

# B-meson decay constants and mixing on the lattice

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## 1 Introduction

It has been argued that differences observed between measurements of some flavor observables and the corresponding Standard Model (SM) predictions may be due to beyond the SM (BSM) physics affecting the neutral  $B$ -meson mixing processes [1, 2]. Although the most recent analysis seem to indicate that there are not large BSM contribution to neutral  $B$ -meson mixing [3], the future will bring new twists, and precise calculations of the non-perturbative inputs parametrizing the mixing in the SM and beyond are necessary for a thorough understanding of quark flavor physics. Lattice QCD can provide those non-perturbative theoretical inputs from first principles and with errors at the few percent level.

The use of lattice QCD techniques can also shed light on the tension observed between the measured value of  $\mathcal{B}r(B \rightarrow \tau\nu)$  and SM predictions [4], by providing precise values of  $f_B$ . Unitarity Triangle fits are very sensitive to  $f_B$  and processes with potential to reveal NP effects depend on  $f_B$  or  $f_{B_s}$ , so any improvement in the decay constant calculations is very important.

In the next Sections I will summarize the status of calculations of both  $B$ -meson decay constants and non-perturbative inputs relevant for the study of  $B^0 - \bar{B}^0$  mixing using lattice QCD techniques. I will only discuss realistic calculations with a complete error budget, simulations at several lattice spacings (and an extrapolation to the continuum), and including three dynamical quark flavors ( $N_f = 2 + 1$  simulations).

## 2 Decay constants

Simulating heavy quarks on the lattice implies having to deal with discretization errors entering in powers of the mass in lattice units,  $am_Q$ . These corrections are not negligible at typical lattice spacings  $a$ . In particular, the  $b$  quark can not be simulated at its physical mass on present lattices even with improved actions, since, typically,  $am_b > 1$ . There are two approaches that have been used to solve this

problem: simulating the heavy quarks with effective theories (such as heavy-quark effective theories or non-relativistic QCD), and performing relativistic simulations with improved actions but with smaller masses than the physical bottom mass (but the same order or larger than the charm mass) and then extrapolating those results up to the physical  $m_b$ . The first approach is being used by the Alpha (heavy-quark effective theory, HQET), HPQCD (non-relativistic QCD, NRQCD), FNAL/MILC (Fermilab action), and RBC/UKQCD (non-perturbatively relativistic heavy-quark action) Collaborations. One of the dominant systematic errors in these calculations is the one associated with the use of an effective theory. The collaborations that follow a relativistic approach for  $b$  quarks and whose results I will mention here are ETMC (tm action) and HPQCD (HISQ action).

There have been three lattice  $N_f = 2 + 1$  calculations of  $f_B$  and  $f_{B_s}$  in the last two years by the HPQCD [5, 6] (with a relativistic and a non-relativistic approach respectively) and the FNAL/MILC [7] collaborations, which have reduced the error to the 2.5% level. The smallest error in [6] for  $f_B$  is achieved by determining the ratio  $f_{B_s}/f_B$  using a non-relativistic description (NRQCD) of the  $b$  quarks together with the  $f_{B_s}$  determination in [5], which employs relativistic  $b$  quarks. By using the ratio  $f_{B_s}/f_B$  the dominant systematic errors associated with the effective NRQCD description partially cancel, so this determination is nearly free of uncertainties due to an effective description of the  $b$  quark. The average values of  $f_B$  and  $f_{B_s}$  from these three calculations are [8]

$$f_B = 190.6(4.7)\text{MeV}, \quad f_{B_s} = 227.6(5.0)\text{MeV}, \quad \text{and} \quad \frac{f_{B_s}}{f_{B_d}} = 1.201(17). \quad (1)$$

The direct comparison of the results in Eq. (1) with experiment is problematic due to the need of the value of the CKM matrix element  $|V_{ub}|$  (whose inclusive and exclusive determinations disagree at the  $3\sigma$  level) and the  $\sim 2\sigma$  disagreement of BaBar [9] and Belle's [10] measurements. Nevertheless, Belle new measurement seems to alleviate the tension between theory and experiment.

In the next two years there will be new results for  $B$ -meson decay constants with  $N_f = 2 + 1$  and  $N_f = 2 + 1 + 1$  from lattice collaborations using both an effective theory description (RBC/UKQCD [11]) and a relativistic description (ETMC <sup>1</sup>), as well updates with considerably smaller errors from FNAL/MILC [13].

## 2.1 Neutral $B$ -meson mixing

The current status of  $N_f = 2 + 1$  lattice calculations of the non-perturbative quantities parametrizing the mass differences between the heavy and the light mass eigenstates in both the  $B_d^0$  and  $B_s^0$  systems, as well as the SU(3) breaking ratio  $\xi$ , is summarized

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<sup>1</sup> Their  $N_f = 2$  results [12] agree within  $1\sigma$  with Eq. (1).

	HPQCD	FNAL/MILC	RBC/UKQCD
$\xi$	1.258(33)	1.27(6)	1.13(12)
$B_{B_s}/B_{B_d}$	1.05(7)	1.06(11)	-
HPQCD: $f_{B_s}\sqrt{\hat{B}_{B_s}} = 266(6)(17)$ MeV, $\hat{B}_{B_s} = 1.33(6)$			
HPQCD: $f_{B_d}\sqrt{\hat{B}_{B_d}} = 216(9)(13)$ MeV, $\hat{B}_{B_d} = 1.26(11)$			

Table 1:  $B$ -meson mixing parameters.  $\xi$  is defined as the ratio of the parameters in the second and third rows. In the case where there are two errors, the first one is statistical and the second one systematic.

in Tab. 1. The HPQCD [14] and FNAL/MILC [15] collaborations use the same light quark formulation, but a different description for the  $b$  quarks. The exploratory study by the RBC/UKQCD collaboration uses heavy quarks in the static limit [16]. The average of the results in Tab. 1 for  $\xi$  gives the value  $\xi = 1.251 \pm 0.032$ . The calculations whose results are shown in Tab. 1 were not optimized to extract the bag parameters but rather the matrix elements, so the errors for the bag parameters can be significantly reduced to about a 3% error in on-going calculations.

There is not yet a finalized calculation of the matrix elements needed for the determination of the decay width differences,  $\Delta\Gamma_{d,s}$ , in the continuum limit and with  $N_f = 2 + 1$  flavors of sea quarks, but preliminary results for the relevant matrix elements by FNAL/MILC can be found in [17].

Beyond the SM the mixing parameters can have contributions from  $\Delta B = 2$  four-fermion operators which do not contribute in the SM. The matrix elements of the five operators in the complete basis describing  $\Delta B = 2$  processes, together with the Wilson coefficients for those operators calculated in a particular BSM theory and the experimental measurements of the mixing parameters, can provide very useful constraints on that BSM theory. Again, there is not a final unquenched lattice calculation of the matrix elements of all the operators in the  $\Delta B = 2$  effective hamiltonian, but FNAL/MILC presented preliminary results for the complete basis in [17].

The authors of Ref. [18] suggested that the branching fractions of the rare decays  $B_q \rightarrow \mu^+\mu^-$  (for  $q = s, d$ ) could be determined from the experimental measurement of the mass difference in the neutral  $B_q$ -meson system,  $\Delta M_q$ , and the lattice calculation of the bag parameter  $\hat{B}_{B_q}$  using

$$\frac{\mathcal{B}r(B_q \rightarrow \mu^+\mu^-)}{\Delta M_q} = \tau(B_q) 6\pi \frac{\eta_Y}{\eta_B} \left( \frac{\alpha}{4\pi M_W \sin^2\theta_W} \right)^2 m_\mu^2 \frac{Y^2(x_t)}{S(x_t)} \frac{1}{\hat{B}_q}. \quad (2)$$

In order to compare experimental measurements and the theory predictions for the decay rate of  $B_s^0$ , one must include the effects of a non-vanishing  $\Delta\Gamma_s$  [19]. This can be done in the SM by rescaling the theory prediction by  $1/(1 - y_s)$ , where  $y_s \equiv$

$\tau_{B_s} \Delta\Gamma_s/2$  [19]. Multiplying Eq. (2) by this factor for the  $B_s^0 \rightarrow \mu^+\mu^-$  decay and using the HPQCD determination of the bag parameters  $\hat{B}_{B_s} = 1.33(6)$  and  $\hat{B}_{B_d} = 1.26(11)$  [14]; together with  $\tau_{B_s} = 1.497(15)\text{ps}$ ,  $\tau_{B_d} = 1.519(7)\text{ps}$  [20], and  $\Delta\Gamma_s = 0.116(19)\text{ps}^{-1}$  [21], one gets

$$\begin{aligned}\mathcal{B}r(B_s \rightarrow \mu^+\mu^-)|_{y_s} &= (3.65 \pm 0.20) \times 10^{-9}, \\ \mathcal{B}r(B_d \rightarrow \mu^+\mu^-) &= (1.04 \pm 0.09) \times 10^{-10}.\end{aligned}\tag{3}$$

The direct calculation of these branching fractions has become competitive with the one in (3) [22] thanks to the recent improvements in the calculation of the  $B$ -meson decay constants on the lattice summarized in Sec. 2. Including the correction factor  $1/(1 - y_s)$  for the  $B_s \rightarrow \mu^+\mu^-$  decay rate, and using the same inputs as in Ref. [22] except for  $f_B$  and  $f_{B_s}$ , for which I use the averages described in Sec. 2, and  $\tau_{B_s}$ , which I take equal to its PDG 2012 value,  $\tau_{B_s} = 1.497(15)\text{ps}$ , I get

$$\begin{aligned}\mathcal{B}r(B_s \rightarrow \mu^+\mu^-)|_{y_s} &= (3.64 \pm 0.23) \cdot 10^{-9}, \\ \mathcal{B}r(B_d \rightarrow \mu^+\mu^-) &= (1.07 \pm 0.10) \cdot 10^{-10}.\end{aligned}\tag{4}$$

The agreement between the two set of numbers in (3) and (4) is excellent. This gives us confidence in the SM prediction for these branching fractions, and this confidence will increase when we have results for the bag parameters entering in (3) from the on-going lattice calculations described above. This is very important since the LHC bounds are now approaching the SM predictions <sup>2</sup>, especially for  $B_s^0$  decays,  $\mathcal{B}r(B_s \rightarrow \mu^+\mu^-)|_{LHC} < 4.2 \cdot 10^{-9}$  at 95% CL [24].

### 3 Outlook and future prospects

In the near future there will be results for the  $B$ -meson mixing parameters and the decay constants at the percent level and with different lattice formulations for both light and heavy quarks. Some of the results expected for next year are: the first unquenched determinations in the continuum limit of matrix elements needed for the calculation of the decay width differences  $\Delta\Gamma_{s,d}$ , the analysis of  $B$  mixing in BSM theories, and the study of short-distance contributions to  $D^0 - \bar{D}^0$  mixing.

Further improvement will be achieved by using simulations at the physical  $u$  and  $d$  masses (some preliminary results for  $f_B$  are already available from the HPQCD collaboration), including the effect of dynamical charm quarks, and extending the use of relativistic actions to the analysis of  $B^0 - \bar{B}^0$  mixing.

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<sup>2</sup>After this conference, the LHCb Collaboration announced the first evidence for one of these two processes [23],  $\mathcal{B}r(B_s \rightarrow \mu^+\mu^-) = (3.2_{-1.2}^{+1.5}) \cdot 10^{-9}$ . This result agrees with the SM prediction given in this proceeding.

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